Math 121   1.2 Exponents

Objectives

1) Use the properties of exponents to simplify expressions.
2) Evaluate roots + radicals, including fraction exponents
3) Evaluate negative and zero exponents
4) Simplify expressions combining any of the above
5) Avoid common errors with square roots and squares.

Honors 6) Power Regression

Exponent Laws

Exponent law #1 $a^n \cdot a^m = a^{n+m}$ Example: $x^2 \cdot x^3 = x^{2+3} = x^5$

With fraction exponents, it works the same

- Same base, appears twice
- Multiplied bases
- Add exponents

$$a^{p/q} \cdot a^{r/t} = a^{p/q + r/t}$$

Fraction example: $x^{2/3} \cdot x^{1/6} = x^{2/3 + 1/6} = x^{5/6}$

<u>Caution</u>: Be patient! You know how to add fractions, but it may take an extra step of work, or careful use of your calculator.

Exponent law #2 $\dfrac{a^n}{a^m} = a^{n-m}$ Examples: $\dfrac{x^3}{x^2} = x^{3-2} = x^1 = x$

$$\frac{x^4}{x^7} = \frac{1}{x^{7-4}} = \frac{1}{x^3} = x^{-3}$$

With fraction exponents, it works the same

- Same base, appears twice
- Divided bases
- Subtract exponents, numerator exponent minus denominator →result in numerator
- OR Subtract exponents, denominator exponent minus numerator →result in denominator

$$\frac{a^{p/q}}{a^{r/t}} = a^{p/q - r/t}$$

Fraction examples: $\dfrac{x^{2/3}}{x^{1/6}} = x^{2/3 - 1/6} = x^{3/6} = x^{1/2}$

$$\frac{x^{1/4}}{x^{3/4}} = \frac{1}{x^{3/4 - 1/4}} = \frac{1}{x^{2/4}} = \frac{1}{x^{1/2}}$$

Exponent law #3 $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$ Examples: $x^{-3} = \dfrac{1}{x^3}$

$$\frac{1}{x^{-3}} = x^3$$

With fraction exponents, it works the same

- Negative exponent
- Move exponent from numerator to denominator (or denominator to numerator) of fraction, change sign of exponent to positive

$$a^{-p/q} = \frac{1}{a^{p/q}}$$

Fraction example: $x^{-2/3} = \dfrac{1}{x^{2/3}}$

$$\frac{1}{a^{-p/q}} = a^{p/q}$$ Fraction example: $$\frac{1}{x^{-2/3}} = x^{2/3}$$

Exponent law #4 $a^0 = 1$ Examples: $1 = \frac{x^2}{x^2} = x^{2-2} = x^0 = 1$

Fraction example: $1 = \frac{x^{2/3}}{x^{2/3}} = x^{2/3 - 2/3} = x^0 = 1$

Caution: Zero exponent resolves to the number 1. There is no base anymore!

Exponent law #5 $(a^n)^m = a^{n \cdot m}$ Example: $(x^{-3})^2 = x^{-3 \cdot 2} = x^{-6} = \frac{1}{x^6}$

With fraction exponents, it works the same

- One base
- Parentheses separate two exponents
- Multiply exponents

$$\left(a^{p/q}\right)^{r/t} = a^{p/q \cdot r/t}$$ Fraction example: $$\left(x^{2/3}\right)^{1/6} = x^{2/3 \cdot 1/6} = x^{1/9}$$

Exponent law #6 $(ab)^n = a^n b^n$ Examples: $(xy)^{-2} = x^{-2}y^{-2} = \frac{1}{x^2y^2}$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$ $$\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$ $$\left(\frac{x}{y}\right)^{-3} = \left(\frac{y}{x}\right)^3 = \frac{y^3}{x^3}$$

With fraction exponents, it works the same

- Two bases, inside parentheses
- One exponent outside parentheses
- "Share" exponents, each base inside parentheses gets the exponent outside those parentheses

$$(ab)^{p/q} = a^{p/q} b^{p/q}$$ Fraction examples: $$(xy)^{2/3} = x^{2/3} y^{2/3}$$

$$\left(\frac{a}{b}\right)^{p/q} = \frac{a^{p/q}}{b^{p/q}}$$ $$\left(\frac{x}{y}\right)^{2/3} = \frac{a^{2/3}}{b^{2/3}}$$

$$\left(\frac{a}{b}\right)^{-p/q} = \left(\frac{b}{a}\right)^{p/q} = \frac{b^{p/q}}{a^{p/q}}$$ $$\left(\frac{x}{y}\right)^{-2/3} = \left(\frac{y}{x}\right)^{2/3} = \frac{y^{2/3}}{x^{2/3}}$$

Caution: No math term exists for what we do with these exponents. Do not use the word "distribute", which refers specifically to multiplying a sum, such as $3(2+5) = 3 \cdot 2 + 3 \cdot 5$

Examples (solutions are on typed handout)

Simplify and write answers with positive exponents.

1. a) $x^2 \cdot x^3$

   b) $x^{2/3} \cdot x^{1/6}$

2. a) $\dfrac{x^3}{x^2}$

   b) $\dfrac{x^4}{x^7}$

   c) $\dfrac{x^{2/3}}{x^{1/6}}$

   d) $\dfrac{x^{1/4}}{x^{3/4}}$

3. a) x^{-3}

   b) $\dfrac{1}{x^{-3}}$

   c) $x^{-2/3}$

   d) $\dfrac{1}{x^{-2/3}}$

4. a) $(x^{-3})^2$

   b) $(x^{2/3})^{1/6}$

5. a) $(xy)^{-2}$

   b) $\left(\dfrac{x}{y}\right)^3$

   c) $\left(\dfrac{x}{y}\right)^{-3}$

   d) $(xy)^{2/3}$

   e) $\left(\dfrac{x}{y}\right)^{2/3}$

   f) $\left(\dfrac{x}{y}\right)^{-2/3}$

6. a) x^0

   b) $(y^{-2/3})^0$

A radical is a symbol $\sqrt{\ }$, $\sqrt[3]{\ }$, $\sqrt[4]{\ }$, $\sqrt[n]{\ }$

Example: $\sqrt[3]{x}$ ← entire expression is called a radical

3 is called the index

x is called the radicand

vocabulary

Evaluate.

7) a) $\sqrt{16} = \boxed{4}$ b/c $4^2 = 16$

b) $\sqrt{0} = \boxed{0}$ b/c $0^2 = 0$

c) $\sqrt{1} = \boxed{1}$ $1^2 = 1$

d) $\sqrt{-1}$ = not a real # $= \boxed{i}$ ← In calculus, we want only real numbers!

e) $\sqrt{\frac{25}{49}} = \frac{\sqrt{25}}{\sqrt{49}} = \boxed{\frac{5}{7}}$ $\frac{5^2}{7^2} = \frac{25}{49}$.

8) a) $\sqrt[3]{27} = \boxed{3}$ $3^3 = 27$

b) $\sqrt[3]{-27} = \boxed{-3}$ $(-3)^3 = 27$ parentheses required.

c) $\sqrt[3]{\frac{1}{8}} = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \boxed{\frac{1}{2}}$ $\frac{1^3}{2^3} = \frac{1}{8}$

d) $\sqrt[3]{\frac{64}{27}} = \frac{\sqrt[3]{64}}{\sqrt[3]{27}} = \boxed{\frac{4}{3}}$

9) a) $\sqrt[4]{81} = \boxed{3}$ because $3^4 = 81$

b) $\sqrt[4]{\frac{1}{16}} = \frac{\sqrt[4]{1}}{\sqrt[4]{16}} = \boxed{\frac{1}{2}}$ $\frac{1^4}{2^4} = \frac{1}{16}$

c) $\sqrt[5]{-32} = \boxed{-2}$ $(-2)^5 = -32$

d) $\sqrt[5]{\frac{1024}{243}} = \frac{\sqrt[5]{1024}}{\sqrt[5]{243}} = \boxed{\frac{4}{3}}$ $\frac{4^5}{3^5} = \frac{1024}{243}$

You can find radicals on your GC

$\sqrt{\ }$ square roots [2nd] [x^2]

$\sqrt[3]{\ }$ [MATH] 4. $\sqrt[3]{\ }$

$\sqrt[x]{\ }$ [MATH] 5. $\sqrt[x]{\ }$

* Type the index first!

[MATH] 1. ▸frac will convert many decimals to fractions!

$\boxed{\text{CAUTION}}$ "Square roots" vs. "Principal square roots"

"Square roots"	"Principal square roots"
solutions to	Evaluate
Solve. $x^2 = 9$	$x = \sqrt{9}$ or $9^{1/2}$
$\boxed{x = +3, -3}$	$\boxed{x = 3 \text{ ONLY}}$

When we use a radical symbol, we mean only the positive, or principal square root.

Fraction Exponents: $x^{1/n}$

But why is $\sqrt{9} = 9^{1/2}$?

Let's go back to exponents....

$$(x^{1/2})^2 = x^{1/2 \cdot 2} = x^1 = x$$

So... $x^{1/2} \cdot x^{1/2} = x$

If we use a number for x: $9^{1/2} \cdot 9^{1/2} = 9$

This must mean $9^{1/2} = 3 = \sqrt{9}$.

What if it's exponent $1/3$?

$$(x^{1/3})^3 = x^{1/3 \cdot 3} = x^1 = x$$

So $x^{1/3} \cdot x^{1/3} \cdot x^{1/3} = x$

If we use a number for x: $27^{1/3} \cdot 27^{1/3} \cdot 27^{1/3} = 27$

This must mean $\sqrt[3]{27} \cdot \sqrt[3]{27} \cdot \sqrt[3]{27} = 27$

so $\sqrt[3]{27} = 27^{1/3}$.

$$\boxed{x^{1/n} = \sqrt[n]{x}}$$

The denominator of a fraction exponent is the index of the equivalent radical.

Evaluate

10 a) $\left(\frac{25}{4}\right)^{1/2} = \sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}} = \boxed{\frac{5}{2}}$

b) $\left(\frac{-8}{27}\right)^{1/3} = \sqrt[3]{\frac{-8}{27}} = \frac{\sqrt[3]{-8}}{\sqrt[3]{27}} = \boxed{\frac{-2}{3}}$

Fraction Exponents: $x^{m/n}$

Remember fractions....

Evaluate

11 a) $3 \cdot \frac{1}{5} = \frac{3}{1} \cdot \frac{1}{5} = \boxed{\frac{3}{5}}$

b) $\frac{1}{4} \cdot 5 = \frac{1}{4} \cdot \frac{5}{1} = \boxed{\frac{5}{4}}$

Remember exponent laws....

Simplify 12 a) $(x^3)^{1/5} = x^{3 \cdot 1/5} = \boxed{x^{3/5}}$

b) $(x^{1/4})^5 = x^{1/4 \cdot 5} = \boxed{x^{5/4}}$

Remember fraction exponents ...

Evaluate 13 a) $(-32)^{3/5} = \left[(-32)^{1/5}\right]^3 = \left(\sqrt[5]{-32}\right)^3 = (-2)^3 = \boxed{-8}$

b) $\left(\frac{1}{81}\right)^{5/4} = \left(\left(\frac{1}{81}\right)^{1/4}\right)^5 = \left(\sqrt[4]{\frac{1}{81}}\right)^5 = \left(\frac{1}{3}\right)^5 = \boxed{\frac{1}{243}}$

In general...

$$x^{m/n} = \left(\sqrt[n]{x}\right)^m \text{ or } \sqrt[n]{x^m}$$

The denominator is always the index of the radical.

Putting it together....

Evaluate 14 a) $8^{-2/3} = \frac{1}{8^{2/3}}$ negative exponent first!

$= \frac{1}{(\sqrt[3]{8})^2}$ what does the fraction exponent mean? denominator = index of radical

$= \frac{1}{(2)^2}$

$= \boxed{\frac{1}{4}}$

b) $\left(\frac{9}{4}\right)^{-3/2} = \left(\frac{4}{9}\right)^{3/2}$ negative exponent first

$= \left(\sqrt{\frac{4}{9}}\right)^3$ what does fraction exponent mean? denom is index.

$= \left(\frac{2}{3}\right)^3$

$= \frac{2^3}{3^3}$

$= \boxed{\frac{8}{27}}$

CAUTIONS:

NO $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$ ex $\sqrt{9+16} \neq \sqrt{9} + \sqrt{16}$, $5 \neq 3+4$

NO $\sqrt{x-y} \neq \sqrt{x} - \sqrt{y}$ ex $\sqrt{25-16} \neq \sqrt{25} - \sqrt{16}$, $3 \neq 5-4$

YES $\sqrt{x \cdot y} = \sqrt{x} \cdot \sqrt{y}$

YES $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

} so long as $\sqrt{x}$ and $\sqrt{y}$ are real numbers

NO $(x+y)^2 \neq x^2 + y^2$ $(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$

NO $(x-y)^2 \neq x^2 - y^2$ $x^2 - y^2 = (x-y)(x+y)$

YES $(xy)^2 = x^2y^2$

YES $\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$

} exponent laws for multiply + divide ONLY, NOT add + subtract

(15) The following table shows worldwide sales for semiconductors used in cellphones and laptop computers for these years.

Year	2011	2012	2013
Sales in billions \$	80.2	87.2	93.6

a) Let 2011 $\Rightarrow x=1$, and use power regression to fit a curve to this data. Write the model using 4 decimal places.

b) Use your model to predict sales in 2020.

a) Graphing Calculator:

$\{1, 2, 3\} \to L_1$

$\underbrace{\boxed{\text{2nd}}\ \boxed{(}}_{\{}$ 1 $\boxed{,}$ 2 $\boxed{,}$ 3 $\underbrace{\boxed{\text{2nd}}\ \boxed{)}}_{\}}$ $\boxed{\text{STO>}}$ $\underbrace{\boxed{\text{2nd}}\ 1}_{L_1}$.

$\{80.2, 87.2, 93.6\} \to L_2$

$\boxed{\text{2nd}}\ \underbrace{\boxed{(}}_{\{}$ 80.2 $\boxed{,}$ 87.2 $\boxed{,}$ 93.6 $\underbrace{\boxed{\text{2nd}}\ \boxed{)}}_{\}}$ $\boxed{\text{STO>}}$ $\underbrace{\boxed{\text{2nd}}\ 2}_{L_2}$

PwrReg
$y = a*x^b$
$a = 79.91211001$
$b = .1383157555$

$\boxed{\text{STAT}}$ $\triangleright$

EDIT $\boxed{\text{CALC}}$ TESTS
6.
7.
8.
9.
0.
$\boxed{\text{A}}$ PwrReg

$\triangledown$ to next screen

select $\boxed{\text{A}}$ by pressing $\boxed{\text{ENTER}}$.

(or $\boxed{\text{ALPHA}}$ $\boxed{\text{MATH}}$ (A) at any time)

$a \approx 79.9121$
$b \approx .1383$

$$\boxed{y = 79.9121\,x^{.1383}}$$

b) Graphing calculator:

$y_1 =$ RegEQ to fill in all decimal places

$\boxed{\text{Y=}}$ $\boxed{\text{CLEAR}}$ (if necessary)

$\boxed{\text{VARS}}$

$\boxed{\text{VARS}}$ Y-VARS
$\boxed{5}$ Statistics

$\triangleright\ \triangleright$ Twice

XY Σ $\boxed{\text{EQ}}$
$\boxed{1}$ RegEQ

Find x: $2020 - 2011 = 9$

Substitute $x=9$ using TABLE or Y-VARS.

$\boxed{\text{2nd}}$ $\boxed{\text{WINDOW}}$ (TBLSET)

Independent Auto $\boxed{\text{Ask}}$

$\boxed{\text{2nd}}$ $\boxed{\text{GRAPH}}$ (TABLE) 9

X	Y_1
9	108.29

$\boxed{\text{In 2020, predict \$108.29 billion in semiconductor sales}}$